The Precise Evaluation of the Stiffness of Square Threaded Screws

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This paper presents a study on the precise evaluation of the stiffness of square threaded screws which are used in industry. The amount and manner of distribution of stresses that occur in a screw depends on the efforts but also the cross section (shape and size), section where is calculated the stress. The screws are made in many constructive forms and in fuctioning they are calculated at different stresses such as: twisting, bending, shire, buckling or compound stresses to determine the maximum stress of the screw material. These calculations are necessary for establish the functioning conditions for the screws that are used in industry and also for determine the efficiency of the material that are made of.

Keywords: square screws, stiffness, circular sectors, cross section.

In the calculation of every bolt resistance involving knowledge tensile stiffness, bending and twisting, EA, EI_z, GI_p, is usually taken into account, in covering, the inner diameter of the thread [1-2]. But there is an important caveat of the cross section which in reality is not circular but is composed of two circular sectors with geometrical dimensions depending on screw geometry like we will see in this paper.

Experimental part

We consider that in figure 1a from below have an outer diameter of a screw d_e, the inside diameter d_i, the pitch p of the thread and the thickness t, measured along the generator. The cross section will be the one in figure 1b.

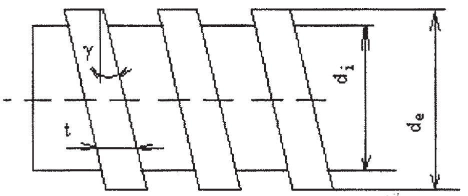

Fig. 1a The square threaded screw

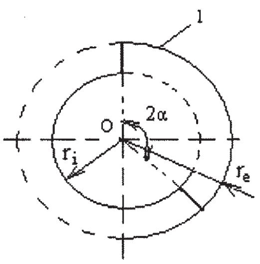

Fig 1b Cross section in a square threaded screw

The length of the large sector of a circle of radius $r_e = d_e/2$, is written with the help of figure 1a such as [3]:

$$l = \frac{t}{tg\gamma} = \frac{t}{\frac{p}{\pi d_e}} = \frac{\pi d_e t}{p} \qquad (1)$$

where p is the pitch screw.

The sector angle 2α and his half will be given by:

$$2\alpha = \frac{l}{r_e} = \frac{\pi d_e t}{r_e p} = 2\pi \frac{t}{p}; \alpha = \pi \frac{t}{p} \qquad (2)$$

The cross section of the screw admits symmetry axis Ox which is the main axis of inertia and the axis perpendicular Oy will be the second major axis like in figure 2 from below:

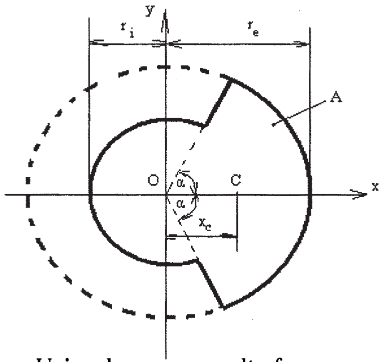

Fig. 2 The cross section of the screw with Ox and Oy main axis of inertia

Using known results from geometry and inertia tables for moments of flat surfaces and centers of gravity tables will be written [1]:

$$A = R^2\alpha; I_{x,y} = \frac{R^4}{8}(2\alpha \pm \sin 2\alpha) \qquad (3)$$

A cross section will be made in accordance with figure 2 from above an will be:

$$A = r_e^2\alpha + r_i^2(\pi - \alpha) \qquad (4)$$

and abscissa x_c of the center of gravity that has the expression:

$$x_c = \frac{\Sigma x_i A_i}{\Sigma A_i} = \frac{2}{3}\frac{r_e^3 \sin\alpha - r_i^3 \sin(\pi - \alpha)}{r_e^2\alpha - r_i^2(\pi - \alpha)} \qquad (5)$$

The moments of inertia of the principal axes of inertia I_x and I_y will be [4]:

$$I_x = \frac{r_e^4}{8}(2\alpha - \sin 2\alpha) + \frac{r_i^4}{8}[2(\pi - \alpha) - \sin 2(\pi - \alpha)]$$

$$I_y = \frac{r_e^4}{8}(2\alpha + \sin 2\alpha) + \frac{r_i^4}{8}[2(\pi - \alpha) + \sin 2(\pi - \alpha)] \qquad (6)$$

The moments of inertia of the principal central axes of inertia will be expressed by:

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$$I_{x_c} = I_x = \frac{r_e^4}{8}(2\alpha - \sin 2\alpha) + \frac{r_i^4}{8}[2(\pi-\alpha) - \sin 2(\pi-\alpha)]$$

$$I_{y_c} = I_y + x_c^2 A = \frac{r_e^4}{8}(2\alpha + \sin 2\alpha) + \frac{r_i^4}{8}[2(\pi-\alpha) + \sin 2(\pi-\alpha)] + \quad (7)$$

$$\left[\frac{2}{3}\frac{r_e^3 \sin\alpha - r_i^3 \sin(\pi-\alpha)}{r_e^2\alpha - r_i^2(\pi-\alpha)}\right]^2 \left[r_e^2\alpha + r_i^2(\pi-\alpha)\right]$$

Corresponding module of resistance will have the following expression:

$$W_x = \frac{I_{X_C}}{r_e \sin\alpha} = \frac{r_e^4}{8r_e \sin\alpha}(2\alpha - \sin 2\alpha) + \frac{r_i^4}{8r_e \sin\alpha}[2(\pi-\alpha) + \sin 2(\pi-\alpha)]$$

$$W'_{y_C} = \frac{I_{y_C}}{x_C + r_i} = \frac{1}{x_c + r_i}\left\{ \frac{r_e^4}{8}(2\alpha + \sin 2\alpha + \frac{r_i^4}{8}[2(\pi-\alpha) + \sin 2(\pi-\alpha)] + \right.$$

$$\left. + \left[\frac{2}{3}\frac{r_e^3 \sin\alpha - r_i^3 \sin(\pi-\alpha}{r_e^2\alpha - r_i^2(\pi-\alpha)}\right]^2 \left[r_e^2\alpha + r_i^2(\pi-\alpha)\right] \right\} \quad (8)$$

$$W''_{y_c} = \frac{I_{y_c}}{r_e - r_c} = \frac{1}{r_e - x_c}\left\{ \frac{r_e^4}{8}(2\alpha + \sin 2\alpha\ ) + \frac{r_i^4}{8}[2(\pi-\alpha) + \sin 2(\pi-\alpha)] + \right.$$

$$\left. + \left[\frac{2}{3}\frac{r_e^3 \sin\alpha - r_i^3 \sin(\pi-\alpha}{r_e^2\alpha - r_i^2(\pi-\alpha)}\right]^2 \left[r_e^2\alpha + r_i^2(\pi-\alpha)\right] \right\}$$

where x_c having the expression (5).

It can be calculated immediately the polar moment of inertia to O:

$$I_o = I_x + I_y = \frac{r_e^4}{2}\alpha + \frac{r_i^4}{2}(\pi-\alpha) \quad (9)$$

and also compared to the center of gravity C:

$$I_x = I_{x_C} + I_{y_C} = \frac{r_e^4}{2}\alpha + \frac{r_i^4}{2}(\pi-\alpha) +$$

$$+ \left[\frac{2}{3}\frac{r_e^3 \sin\alpha - r_i^3 \sin(\pi-\alpha)}{r_e^2\alpha - r_i^2(\pi-\alpha)}\right] \left[r_e^2\alpha - r_i^2(\pi-\alpha)\right] \quad (10)$$

Results and discussions

Using relations (4), (6), (8) and (10) in which á is expressed in radians, can perform accurate calculations of the modules of rigidity EA, EI_x, EI_y, GI_p and also resistance modules W_x and W_y which are involved in the calculations of stiffness and strength of the screws. Modules are interested, as it is known from static stability problems (buckling) and the vibrations of helical screws regarded as straight bars [4]. For having terms of comparison we will analyze the particular case $\alpha = \pi/2$ which implies, according to the relation (1), p = 2t and then we will obtain:

$$A = \frac{\pi}{2}(r_e^2 + r_i^2) \quad x_c = \frac{4}{3\pi}\frac{r_e^3 - r_i^3}{r_e^2 - r_i^2}$$

$$I_{x_C} = I_x = \frac{8}{\pi}(r_e^4 + r_i^4)$$

$$I_{Y_C} = \frac{\pi}{8}(r_e^4 + r_i^4) + \left(\frac{3}{4\pi}\frac{r_e^3 - r_i^3}{r_e^2 - r_i^2}\right)\frac{\pi}{2}(r_e^2 + r_i^2)$$

$$w_{x_c} = \frac{I_{x_C}}{r_e} = \frac{\pi}{8r_e}(r_e^4 + r_i^4)$$

$$w'_{y_C} = \frac{I_{y_C}}{r_e + r_c} = \frac{\frac{8}{\pi}(r_e^4 + r_i^4) + \frac{\pi}{2}(r_e^2 + r_i^2)\left(\frac{4}{3\pi}\frac{r_e^3 - r_i^3}{r_e^2 - r_i^2}\right)}{r_i + \frac{4}{3\pi}\frac{r_e^3 - r_i^3}{r_e^2 + r_i^2}}$$

$$w'_{y_C} = \frac{I_{y_C}}{r_e - x_c} = \frac{\frac{8}{\pi}(r_e^4 + r_i^4) + \frac{\pi}{2}(r_e^2 + r_i^2)\left(\frac{4}{3\pi}\frac{r_e^3 - r_i^3}{r_e^2 - r_i^2}\right)^2}{r_i - \frac{4}{3\pi}\frac{r_e^3 - r_i^3}{r_e^2 - r_i^2}} \quad (11)$$

$$I_0 = \frac{4}{\pi}(r_e^4 + r_i^4), \qquad I_c = \frac{4}{\pi}(r_e^4 - r_i^4) + \frac{\pi}{2}(r_e^2 + r_i^2)\frac{3}{4\pi}\left(\frac{r_e^2 - r_i^3}{r_e^2 - r_i^2}\right)^2$$

In case of hinged bar, critical buckling force is expected to determine with the following relationship [1]:

$$f_{cr} = \eta\frac{\pi^2 EI_{x_c}}{l^2} \quad (12)$$

where η is dimensionless and it has the expression:

$$\eta = \frac{2}{1 + \frac{EI_{y_C}}{EI_{x_C}}} \quad (13)$$

the EI_{xc} is the minimum stiffness like in the following formula:

$$EI_{x_C} = \frac{8}{\pi}(r_e^4 + r_i^4)$$

With this will result for η:

$$\eta = \frac{2}{1 + \frac{\frac{8}{\pi}(r_e^4 + r_i^4)}{\frac{8}{\pi}(r_e^4 + r_i^4) + \frac{\pi}{2}(r_e^2 + r_i^2)\frac{3}{\pi}\left(\frac{r_e^3 - r_i^3}{r_e^2 - r_i^2}\right)^2}}$$

For $r_e/r_i = 1.1$ ratio, is achieved $\eta = 1.7196$ resulting an increase of the critical buckling force 71% greater than that which would calculate with the minimum bending stiffness of the bar after normal buckling calculation.

And if it would make the calculation of square threaded bolt resistance, considering the inner diameter section as a section resistance, will get a flexural capacity less 46% for $r_e/r_i = 1$, as result from the following ratio:

$$\frac{\frac{\pi}{8}\left(r_e^4 + r_i^4\right)}{\pi \frac{r_i^4}{4}} = 2.46$$

Conclusions

Similar reveals a significant increase of flexural stiffness of the screws which implies higher frequencies of their vibrations and increased resistance modules to bending and twisting leading to higher real capacity of taking bending and twisting moments. For cases of bars can be calculated as above the percentages increase.

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Manuscript received: 22.12.2014